Calculus Warm Up #3-3

1. Use the limit definition of the derivative to find f′(x).

$$f(x) = \frac{1}{x + 5}$$

2. Use the alternate limit form of the derivative to find the slope of f(x) when x = 2

$$f(x) = \sqrt{2x - 3}$$

HW Questions: p. 111

In Exercises 1–6, find the average rate of change of the given function over the indicated interval. Compare this average rate of change to the instantaneous rates of change at the endpoints of the interval.

Function	*Interval*
1. $f(t) = 2t + 7$	[1, 2]
3. $f(x) = \frac{1}{x + 1}$	[0, 3]
5. $f(t) = t^2 - 3$	[2, 2.1]

7. The height s at time t of a silver dollar dropped from the World Trade Center is given by $s(t) = -16t^2 + 1350$, where s is measured in feet and t is measured in seconds [$s'(t) = -32t$].
   (a) Find the average velocity on the interval [1, 2].
   (b) Find the instantaneous velocity when $t = 1$ and $t = 2$.
   (c) How long will it take the dollar to hit the ground? $\rightarrow 0$
   (d) Find the velocity of the dollar when it hits the ground.

In Exercises 9–14, use the following position and velocity functions for free-falling objects.

$$s(t) = -16t^2 + v_0 t + s_0$$

$$s'(t) = -32t + v_0$$

9. A projectile is shot upward from the surface of the earth with an initial velocity of 384 feet per second. What is its velocity after 5 seconds? After 10 seconds?

11. A pebble is dropped $\rightarrow V_0 = 0$ from a height of 600 feet $\rightarrow S_0 = 600$. Find the pebble's velocity when it hits the ground.
$\rightarrow V(t) = -32t + 0 \rightarrow$ Need time when hits the ground:
$0 = -16t^2 + 600$
$t = \sqrt{\frac{600}{16}}$
plug into $V(t)$

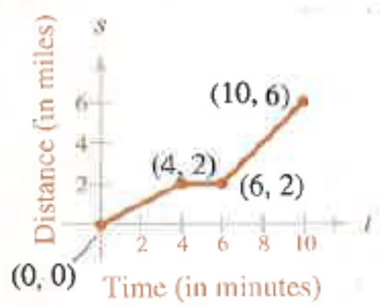
13. To estimate the height of a building, a stone is dropped from the top of the building into a pool of water at ground level. How high is the building if the splash is seen 6.8 seconds after the stone is dropped?

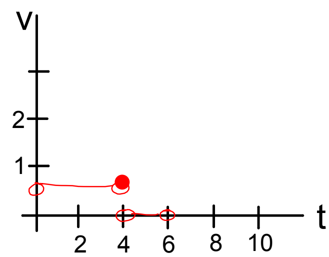
$t = 6.8$ for $s(t) = 0$ $\qquad V_0 = 0$

$0 = -16(6.8)^2 + 0t + S_0$

In Exercises 15 and 16, the graphs of position functions are given. They represent the distance in miles that a person drives during a 10-minute trip to work. Make sketches of the corresponding velocity functions.

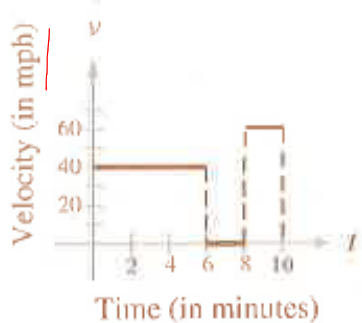
15.

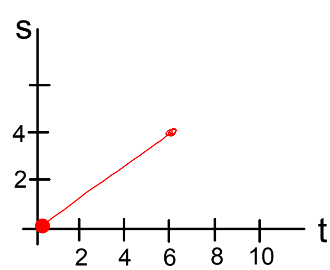




In Exercises 17 and 18, the graphs of velocity functions are given. They represent the velocity in miles per hour during a 10-minute drive to work. Make sketches of the corresponding position functions.

17.





$\frac{40}{60}$ $\frac{2}{3}$

In Exercises 19–24, find the indicated derivative.

Given	*Find*
19. $f'(x) = x^2$	$f''(x)$
21. $f''(x) = 2 - \frac{2}{x}$	$f'''(x)$
23. $f^{(4)}(x) = 2x + 1$	$f^{(6)}(x)$

25. The annual inventory cost for a certain manufacturer is given by

$$C = \frac{1{,}008{,}000}{Q} + 6.3Q$$

$C(350) - C(351)$

where Q is the order size when the inventory is replenished. Find the change in annual cost when Q is increased from 350 to 351 and compare this with the rate of change

$$\frac{dC}{dQ} = -\frac{1{,}008{,}000}{Q^2} + 6.3$$

when $Q = 350$.

27. When a guitar string is plucked, it vibrates with a frequency of

$F = 200\sqrt{T}$ $\longrightarrow$ $F(T) = 200(T)^{1/2}$

where F is measured in vibrations per second and the tension T is measured in pounds. Find the rate of change $\rightarrow F'(T)$ of the frequency when (a) $T = 4$ and (b) $T = 9$.

$$F'(T) = \frac{1}{2}(200)T^{-1/2}$$

$$F'(T) = \frac{100}{\sqrt{T}}$$ ... plug in T's

29. **Newton's Law of Cooling** states that the rate of change of the temperature of an object is proportional to the difference between the object's temperature T and the temperature T_a of the surrounding medium. Write an equation for this law.

$$\Delta T = k(T - T_a)$$

direct variation relationship.

$y = kx$

$\uparrow$

constant of proportionality

3.4 More Differentiation Rules

-Product rule for derivatives

-Quotient rule for derivatives

Product Rule: The product of 2 differentiable functions is itself differentiable.

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + g(x)f'(x)$$

Proof: pg 122

* Notice, the functions take turns!

Example: $f(x) = (3x - 2x^2)(5 + 4x)$

$$f'(x) = (3x-2x^2)\underbrace{(4)}_{\text{derivative of } 2^{nd}\text{ ( )}} + (5+4x)\underbrace{(3-4x)}_{\text{derivative of } 1^{st}\text{ ( )}}$$

Now expand & Simplify

$$f'(x) = 12x - 8x^2 + 15 - 20x + 12x - 16x^2$$

$$f'(x) = -24x^2 + 4x + 15$$

Check using previous method:

Expand first, then use the power rule

$$f(x) = (3x - 2x^2)(5 + 4x)$$

$$f(x) = 15x + 12x^2 - 10x^2 - 8x^3$$

$$f(x) = -8x^3 + 2x^2 + 15x$$

$$f'(x) = -24x^2 + 4x + 15$$

You try:

$$y = (1 + x^{-1})(x - 1)$$

You try:

$y = (1 + x^{-1})(x - 1)$

derivative of 2nd ( )

derivative of 1st ( )

$y' = (1 + x^{-1})(1) + (x-1)(0 + (-1)x^{-2})$

$y' = \frac{x^2}{x^2}\cdot 1 + \frac{1}{x}\cdot\frac{x}{x} + (x-1)\left(-\frac{1}{x^2}\right)$ ← LCD $= x^2$

$y' = \frac{x^2 + x - x + 1}{x^2} \longrightarrow \boxed{y' = \frac{x^2+1}{x^2}}$

Make sure you are not just multiplying by a constant!

Example: $y = 5e(2\pi x^3)$

$y = 10e\pi x^3$

$y' = 30e\pi x^2$:)

What about $\frac{d}{dx}\left[f(x)g(x)h(x)\right]$?

* Just take turns!

$= f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$

Quotient Rule:
The quotient f/g of two differentiable functions f and g is itself differentiable at all values of x for which $g(x) \neq 0$. Moreover, the derivative of f/g is given by the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Proof: pg 124

Lo dee-high minus high dee-lo
over the square of what's below.

dee-high = derivative of the top!

dee-lo = derivative of the bottom.

$$f(x) = \frac{5x - 2}{x^2 + 1}$$

Always start low!!!

Lo dee-high minus high dee-lo

$$\frac{(x^2+1)(5) - (5x-2)(2x)}{(x^2+1)^2}$$

over the square of what's below.

Now simplify: $f'(x) = \dfrac{5x^2 + 5 - 10x^2 - 4x}{(x^2+1)^2}$

$$= \frac{-5x^2 - 4x + 5}{(x^2+1)^2}$$

You try:

1. $y = \dfrac{3 - \frac{1}{x}}{x + 5}$ $\rightarrow 3 - x^{-1}$

Lo dee-high minus high dee-lo

over the square of what's below.

2. $y = \dfrac{x^2 + 3x}{6}$

You try:

1. $y = \dfrac{3 - \frac{1}{x}}{x + 5}$

$y = \dfrac{3 - x^{-1}}{x + 5}$

$y' = \dfrac{(x+5)(0-(-1)x^{-2}) - (3 - x^{-1})(1)}{(x+5)^2}$

$y' = \dfrac{1}{(x+5)^2}\left[(x+5)\left(\frac{1}{x^2}\right) - 3 + \frac{1}{x}\right]$

$y' = \dfrac{1}{(x+5)^2}\left[\frac{x+5}{x^2} - \frac{3x^2}{x^2} + \frac{x}{x^2}\right]$

$y' = \dfrac{-3x^2 + 2x + 5}{x^2(x+5)^2}$

2. $y = \dfrac{x^2 + 3x}{6} \longrightarrow y = \dfrac{x^2}{6} + \dfrac{x}{2}$

$y' = \dfrac{2x}{6} + \dfrac{1}{2}$

$y' = \dfrac{x}{3} + \dfrac{1}{2}$ ☺

☆ Not a quotient of variable expressions!

What about:

1. $y = \dfrac{-3(3x - 2x^2)}{7x}$

First distribute the -3

Choices after that:

* Split the fraction and use power rule

or * use quotient rule

2. $y = \dfrac{9}{5x^2}$ → Rewrite: $y = \frac{9}{5}x^{-2}$

and use Power Rule!

HW: p. 127 # 1-17 eoo, 23, 25 - 37 eoo, 39, 45

Quiz Friday: 3.1-3.3

Limit definition of the derivative

Find derivatives with the power rule

Find the equation of a tangent line